

Answer the following questions. Calculators, Mobile Phones and Pagers are NOT allowed

Find ( 3 points each )

(a)  $\frac{dy}{dt}$  at  $t=9$  given that  $y = \frac{u+2}{u-1}$ ,  $u = (3\sqrt{t}-7)^2$

(b)  $\frac{dy}{dx}$  if  $y$  is implicitly defined by  $x \tan\left(\frac{y}{x}\right) - 1 = 0$ .

(a) ( 4 points ) Let  $f(x) = \begin{cases} \cos x, & x \geq 0 \\ Ax + B, & x < 0 \end{cases}$ . Find  $A$  and  $B$  so that  $f$  is differentiable at 0.

(b) ( 3 points ) Let  $f(x) = \frac{x}{x+1}$ . Find all  $x$  at which the tangent line to the graph of  $f$  is parallel to the line  $4y - x - 3 = 0$ .

(a) ( 3 points ) Let  $f$  and  $g$  be functions for which  $f'(x) = g(x)$  and  $g'(x) = f(x)$  for all  $x$ . Show that  $f^2(x) - g^2(x)$  is a constant function.

(b) ( 4 points ) A ladder 13 ft long is leaning against the side of a building. If the bottom of the ladder is pulled away from the building at a rate of  $\frac{1}{10}$  ft/sec how fast is the angle formed by the ladder and the ground changing at the instant when the top of the ladder is 12 ft above the ground?

Let  $f$  be a differentiable function on  $(-\infty, \infty)$  with

$$f(-2) = -1, \quad f(-1) = -2, \quad f(0) = 0, \quad f(1) = 2, \quad f(2) = 1, \quad \lim_{x \rightarrow \pm\infty} f(x) = 0$$

and

Interval	$(-\infty, -2)$	$(-2, -1)$	$(-1, 0)$	$(0, 1)$	$(1, 2)$	$(2, \infty)$
Sign of $f'(x)$	-	-	+	+	-	-
Sign of $f''(x)$	-	+	+	-	-	+

Answer the following ( 1 point each )

- What are the intervals on which  $f$  is increasing, and the intervals on which  $f$  is decreasing ?
- What are the local extrema of  $f$ , if any ?
- Determine the intervals on which the graph of  $f$  is concave upward and the intervals on which the graph is concave downward.
- What are the points of inflection of the graph of  $f$ , if any ?
- Sketch a graph of  $f$  indicating local extrema, inflection points, concavity, and asymptotes.